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NO. 12227 (LL-153)

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DEVELOPMENT OF A MATHEMATICAL MODEL  
FOR THE PREDICTION OF THE OFF-ROAD  
PERFORMANCE OF 4x4 VEHICLES



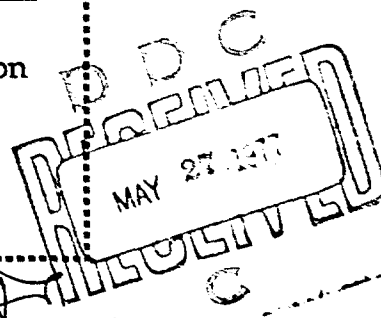
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by Leslie L. Karafiath

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Grumman Aerospace Corporation  
Bethpage, New York 11714

Contract DAAE07-75-C-0066

**U.S. ARMY TANK-AUTOMOTIVE  
RESEARCH AND DEVELOPMENT COMMAND**  
Warren, Michigan 48090



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OF THE OFF-ROAD PERFORMANCE OF 4x4 VEHICLES

Final Report No. 12227 (LL-153)

by

Leslie L. Karafiath

Prepared Under Contract DAAE07-75-C-0066, Amendment P0002

for

United States Army Tank-Automotive Research  
and Development Command  
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January 1977

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Charles E. Mack, Jr.  
Director of Research

## FOREWORD

The improvement of land mobility technology and the determination of future requirements for the mobility of combat and support vehicles of the Army requires an objective evaluation of existing and conceptual vehicles under various terrain, weather, and climatic conditions. The development of the AMC '74 Mobility Model serves this purpose. In this model various submodels simulate the interactions of man-vehicle-terrain systems that determine the overall effectiveness of vehicles in combat and support situations. The off-road performance of wheeled vehicles is computed in the areal module of the AMC '74 Mobility Model where the available tractive force is calculated from empirically established relationships on the assumption that the motion resistance is independent of the driving torque. These relations are available only for either purely frictional or purely cohesive soils.

At Grumman, driven and towed tire-soil interaction models have been developed under earlier contracts with TACOM (now TARADCOM). These models are based on the theoretical concept that soil behavior is characterized by its fundamental (Coulomb) strength parameters and its reactions to tire loads can be determined by plasticity theory methods. The tire-soil interaction models developed on this basis have been validated by test data on a variety of tire sizes and soil conditions; the driven tire-soil model simulates tire performance realistically by accounting for the interaction between driving torque and motion resistance. These tire-soil interaction models are not restricted to purely frictional or purely cohesive soils, and have, therefore, more general applicability than the empirical relationships in the present Mobility Model. Thus, a vehicle model utilizing the tire-soil models is called for to extend the applicability of the AMC-74 Mobility Model to cohesive-frictional soils. The vehicle performance model, prepared under this contract, is described in Section 2 of this report. Additionally, a braked tire model has been developed under the present contract to make the computation of braking performance consistent with the vehicle performance model. Development of the tire-soil interaction model for braked conditions is discussed in Section 3 of this report.

The soil characterization by its Coulomb strength parameters in the vehicle performance model allows the use of the model in practically all types of soils (including extra-terrestrial conditions). Nevertheless, a large data bank exists where the soil properties are characterized by cone index values. To use this information with the vehicle performance model it is necessary to convert cone index values to Coulomb strength

parameters. The differential equations of plasticity for soils, extended for the axially symmetric case of cone penetration, have been applied to this problem. A new method that considers "locked in" stress states in the soil during the process of penetration has been developed for the evaluation of the variation of cone penetration resistance with depth. The results of this research are reported in Section 4.

## ABSTRACT

A mathematical model of 4x4 off-road vehicles has been developed for the estimation of vehicle performance. The model uses the pneumatic-tire soil interaction models developed earlier for driven and towed tires and incorporates these as submodels in the vehicle performance model. Vehicle-soil interactions, such as redistribution of axle weights, due to the slope angle and applied torque, and effect of compaction by the lead wheel are taken into account. The effect on various torque transfer mechanisms between the axles is also considered. The computer program for the vehicle performance model has been prepared as a subroutine with suitable arguments for use in the AMC Mobility Model. The vehicle performance model can be used with any soil, the strength of which can be characterized by its Coulomb strength parameters.

A braked tire-soil interaction model has also been developed for the estimation of the braking force that the vehicle can develop under various soil conditions.

A new method of analysis of the variation of cone penetration resistance with depth has been developed. In this method incremental penetration is analyzed by assuming that the stress state in the soil produced by the previous increment remains "locked in." Cone penetration resistance profiles can be converted to Coulomb strength parameters by this method using a trial and error procedure.

## ACKNOWLEDGEMENT

The work reported herein was performed for the Mobility Systems Laboratory of the U.S. Army Tank-Automotive Research and Development Command (TARADCOM), Warren, Michigan, under the general supervision of Dr. Jack G. Parks, Chief of the Engineering Science Division and Mr. Zoltan J. Janosi, Supervisor, Methodology Function. Mr. Zoltan J. Janosi was also technical monitor. Their help and valuable suggestions in carrying out this work are gratefully acknowledged.

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## 1. SCOPE OF WORK

The scope of work as described in the present supplemental contract agreement includes the following items:

- o Development of a 4 x 4 vehicle performance model using the driven and towed tire models developed at Grumman under earlier contracts. The vehicle performance model should be suitable as an alternate submodel of the AMC '74 Mobility Model.
- o Analysis of braking conditions in soft soils and development of braked tire model to be used as an alternate of the total braking force computation routine in the AMC '74 Mobility Model.
- o Development of relationships for various types of soils between fundamental (Coulomb) soil strength parameters and the soft soil strength data required as input to the AMC '74 Mobility Model.

## 2. THE 4 x 4 VEHICLE PERFORMANCE MODEL

### CONCEPT

For some time it has been recognized that the performance of single tires or rigid wheels cannot be superimposed to obtain that of a wheeled vehicle, since there are interactions among the individual running gears and the vehicle. Instruments of these interactions are the soil, the power train of the vehicle, and its suspension system. This latter affects vehicle performance primarily through its controlling influence on ride dynamics. Since in the AMC '74 Mobility Model (Ref. 1) the effect of ride dynamics on vehicle performance is considered in a separate module, interactions of the suspension system with the running gear assembly have been disregarded in the development of the 4 x 4 vehicle performance model. However, there are interactions between the two axles of a 4 x 4 vehicle that are independent of the suspension. These have been taken into account in the model development in the following way. The load distribution between the front and rear axle, defined by the location of the center of gravity of the vehicle and force equilibrium conditions, changes with the magnitude of grade and the applied driving torque. In the vehicle model this interactive distribution of axle loads is taken into account by computing the actual load on each axle from the equilibrium conditions for each value of the applied driving torque and slope angle.

In four wheel drive vehicles the type of torque transfer mechanisms affect the torque distribution between the axles. A torque transfer coefficient incorporated in the model allows the consideration of an interaxle differential (front and rear axle torques are equal), no interaxle differential (front and rear wheels turn with same speed, respective torques are different due to differences in axle loads), or any torque biased transfer mechanism.

The soil is also an instrument of interaction between the front and rear axle. The leading wheel compacts the soil and thereby changes the properties of soil that control the performance of the trailing wheel. This effect is discussed in more detail in Section 4.

The driven tire-soil interaction model developed under an earlier contract computes tire performance for a given slip value. In the AMC-'74 Mobility Model the "slip modified tractive effort" of a vehicle is computed from the tractive force that the power train of the vehicle can provide at full throttle at maximum, median, and minimum

speed in each gear. To make the vehicle performance model compatible with the AMC '74 Mobility Model it was necessary to make certain modifications in the general scheme of computation of the earlier tire-soil model. This modified computation scheme provides for the computation of the torques supplied by the power train and the determination of the drawbar pull that this torque can develop. The slip associated with this torque is also computed. With this modification the computer program for the vehicle performance model, prepared as a subroutine, is a self-contained alternate for the "slip modified tractive effort" calculation in the areal module of the AMC '74 Mobility Model.

Note that if strict adherence to the structure of the AMC '74 Mobility Model had not been required, another, more economical, use of the vehicle performance model could have been chosen for the determination of the maximum speed that the vehicle can develop under the given terrain conditions. In the AMC '74 Mobility Model relatively simple formulas are used for determination of the slip modified tractive effort, therefore, its computation over the whole range of speed requires little computer time. The actual speed is then determined from the slip modified tractive effort vs speed curve as the maximum at which the tractive effort required under the given conditions is available. The 4 x 4 vehicle performance model uses more elaborate theory and computer techniques for determination of the tractive effort, therefore, savings in computer time could materialize with a different sequence of computations. In this sequence, the external resistances in crossing a terrain unit would be determined first and the vehicle performance model would be used only for the computation of torque necessary to develop an equivalent drawbar pull. The maximum speed of the vehicle would be determined from the computed torque and power train data.

#### MODEL COMPONENTS

The main components of the vehicle performance model are the pneumatic tire-soil interaction models developed for driven and towed tires under previous TACOM contracts. Many computation schemes in these two tire models are common to both, therefore, the computer programs for these two models were integrated in a single computer program for use in the vehicle performance model. In connection with this integration, it was deemed desirable to make a slight modification in the computation of tire performances at low torque. In the driven tire-soil interaction model the interface shear stresses are computed on the assumption that the interface friction angle,  $\delta$ , is uniform over the interface. This assumption is a reasonable one for relatively high torque values. In the towed tire-soil interaction model the interface friction angle,  $\delta$ , is assumed to decrease linearly from a  $\delta_0$  value at the entry angle to zero in the center of the contact area, and

then to decrease further so as to reach  $-\delta_0$  at the rear angle. At low torque the interface shear stress distribution is closer to that at zero torque (towed condition) than the distribution assumed for driven tires. To avoid inconsistencies and abrupt transitions from the zero torque to the low torque condition, the interface shear stress distribution at low torque was modified so as to ensure a gradual transition from the low torque to the towed (zero torque) condition. In the integrated tire-soil performance model, it was assumed that at a torque value corresponding to  $\delta = 0.25 \delta_{\max}$  the zero torque type distribution (variable  $\delta$ ) starts to superimpose over the  $\delta = \text{constant}$  distribution. A gradual transition in the interface shear stress distribution from low torque to zero torque is thereby obtained. Typical interface shear stress distributions resulting from these assumptions are shown in Fig. 1.

#### DESCRIPTION OF VEHICLE PERFORMANCE MODEL

In the development of the vehicle performance model the variable names and other designations of the AMC '74 Mobility Model were adhered to. Nevertheless, because of the additional capabilities of the Grumman vehicle performance model it was necessary to introduce certain new designations and input data information. These are as follows:

##### Vehicle Characteristics:

ITRSF	= 0	No interaxle differential
	= 1	Interaxle differential
	= 2	Torque biased transfer case
TRQFAC	=	1st axle torque/total torque

It is recommended that this information on the vehicle power train characteristics be included in the vehicle data sheet and vehicle preprocessor module of the AMC '74 Mobility Model.

##### Soil Characteristics:

IST	= 9	Designates a cohesive-frictional type soil heretofore not included in the AMC '74 Mobility Model
COHES	=	Cohesion (psi)
PHI	=	Friction angle (degree)
GAMMA	=	Unit weight of soil (lbs/cu in.)
SJ	=	Parameter $j_0$ in shear stress-slip equation
SK	=	Parameter K in shear stress-slip equation

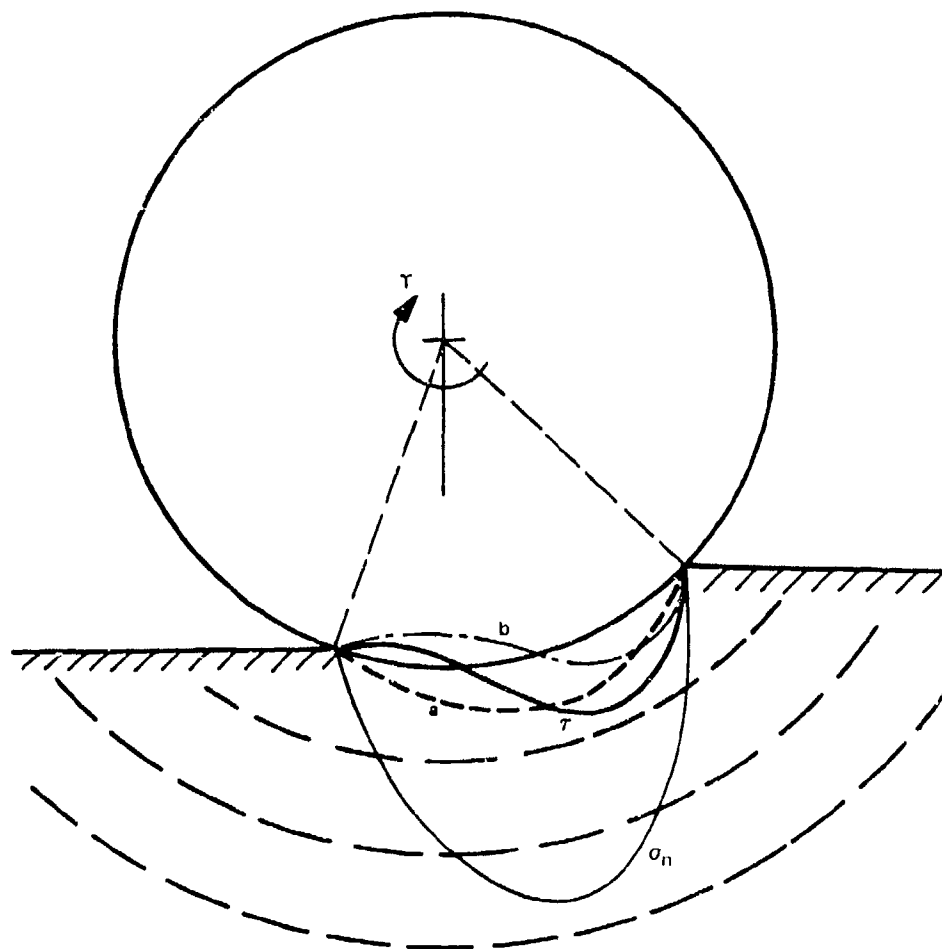


Fig. 1. Distribution of Shear Stress ( $\tau$ ) Beneath a Tire Driven by a Small Torque Results from the Superposition of Shear Stresses from Uniform Interface Friction Angle (a) and for Towed Condition (b).

SFAC =

Parameter  $K_s$  in the estimation of compactive effect of leading wheel. If no value for SAFC has been assigned, a default value of SFAC = 0.9 is used.

It is recommended that these input data for cohesive frictional soils be included in the primary terrain descriptor module.

The computer program for the vehicle performance model is written as subroutine "TIRE" to be called from the AMC '74 Mobility Model as an alternate to the "Slip modified tractive effort" computation in the area submodule. Simple variable, constant, and array designations and dimensions are identical with those used in the AMC '74 Mobility Model (except for the new terms listed before). The subroutine TIRE is called with arguments in the following order:

INPUT VALUES:

ATF (NG) BTF (NG), CTF (NG)	Constant of quadratic fitted to vehicle tractive effort curve in gear NG
WGHT (i)	Weight on axle i
DIAW (i)	Outside wheel diameter of unloaded tires on axle i
SECTW (i)	Section width of tires on axle i
TPSI (i,j)	Tire inflation pressure on axle i, specified for soil type j = IST
DFLECT (i, j)	Deflection of tire on axle i, at pressure specified for j = IST
SECTH (i)	Section height of tire on axle i
VGW (NG, MD)	Mid range, minimum and maximum speed, respectively, in gear NG
VGW (NG, MN)	
VGW (NG, MX)	
IST	Soil type: = 1 for fine grained (cohesive) soil = 2 for coarse grained (frictional) soil = 9 for frictional-cohesive soil
RCIC (j)	Soil Strength (cone index) for j = 1 dry season j = 2 normal season j = 3 wet season

THETA (K)	Slope angle in radians for upslope (K = 1), level (K = 2) and downslope (K = 3)
CGR	Horizontal distance of the center of gravity from rear axle
CGH	Height of center of gravity (for loaded vehicle)
TL	Axle distance
NGR	Number of transmission gear ratios
TRQFAC	See new designations (Pg 4)
ISEAS (i)	Indicator for dry (i = 1) for normal (i = 2) and for wet (i = 3) season
COHES	See new designations (Pg 4)
PHI	See new designations
GAMMA	See new designations
SJ	See new designations
SK	See new designations
IP (i)	= 1 if axle i is powered = 0 otherwise
NTRAV	= 1 for traverse, = 3 for average up, level and down travel
EFC	Elevation correction factor for tractive effort
ITRSF	See new designations (Pg 4)

Note: TPSI (i,j) and DFLECT (i,j) need not be specified for IST = 9 since  
for IST = 9 the values of these tire characteristics have been assumed  
as the average of the values for IST = 1 and 2.

#### OUTPUT VALUES:

VG (NG, IV)	= Speed in gear NG modified by slip (minimum: IV = 1, mid range: IV = 2, maximum: IV = 3)
STRACT (NG, IV, K)	= Slip modified tractive effort in gear NG at minimum (IV = 1), mid range (IV = 1), mid range (IV = 2), and maximum (IV = 3) speed, upslope (K = 1), level (K = 2), and down- slope (K = 3)



FA (NG, K), FB (NG, K)	= Constants for quadratic fitted to slip
FC (NG, K)	modified tractive effort vs speed curve for
	gear NG and slope up (K = 1), level (K = 2),
	and down (K = 3)
FORMX (K)	= Maximum tractive effort available in soil
	for slope up (K = 1), level (K = 2), and
	down (K = 3)
VFMAX (K)	= Speed at which maximum tractive effort
	is available

In the vehicle performance model the slip modified tractive effort is computed from the input data in the following way (Fig. 2).

First, the program checks whether Coulomb soil strength parameters are available. If not, the program computes the Coulomb soil strength parameters from cone index values using the approximate formulas given in Section 4 for cohesive (IST = 1) or frictional soils (IST = 2). Then, a fourfold "Do Loop" is entered for the computation of the slip modified tractive effort for the conditions set forth in the AMC '74 Mobility Model. In this loop the tractive force available from the drive train and the corresponding axle torque is computed first, then the axle loads are determined with respect to the weight redistribution due to the slope angle and applied torque.

The computation of axle performance is started with the axle that carries the lesser load. The reason for this sequence of the computations is that in the case of four wheel drive and interaxle differential the axle torque is limited to whichever of the axle torques is lower. In the case of a powered axle, axle performance computations are performed by following the computation scheme for a driven tire developed under an earlier contract (Ref. 2), while in the case of a free rolling axle the computation follows the scheme for towed tires (Ref. 3). The two computation schemes were integrated in a single program for their expedient use in the vehicle performance model. Details of these computation schemes are given in Refs. (2) and (3). The computation scheme for driven tires determines the drawbar pull and torque values for given load and interface friction angle or slip. In the vehicle performance model the determination of the tractive performance (or drawbar pull) for a given input torque is needed in the case of powered axles. To this end an estimate of the interface friction angle,  $\delta$ , is made and adjusted in an iterative scheme until the computed axle torque agrees with the input torque within the allowed tolerance.

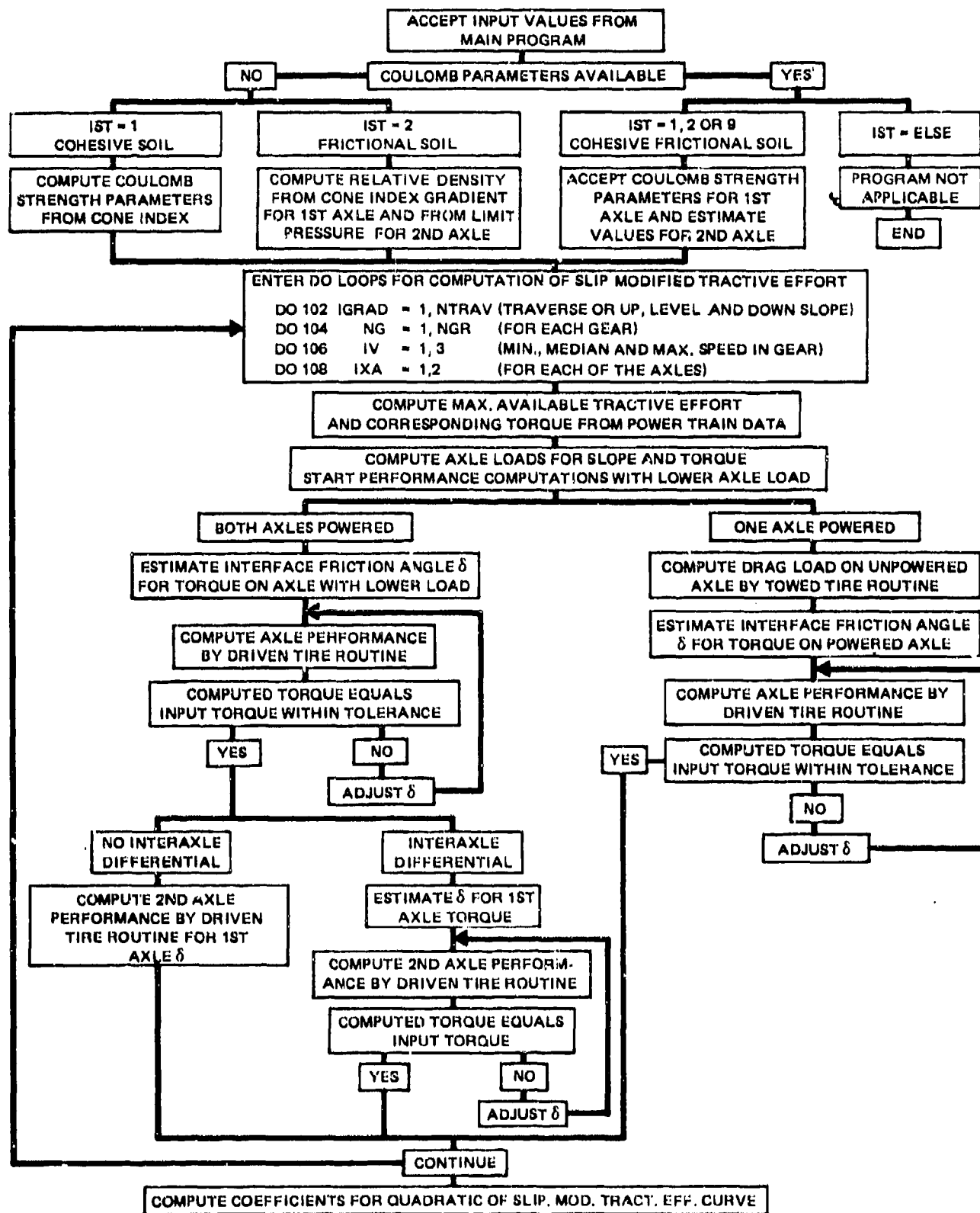


Fig. 2. Flow Diagram for the Computation of Slip Modified Tractive Effort by the Vehicle Performance Model.

In the case of four wheel drive the effect of interaxle differential is taken into account by requiring that the torque on the axle carrying the heavier load be the same as for the axle carrying the lighter load. The tractive performance of the axle under the heavier load is computed on this condition. If the interaxle differential is torque biased then the ratio of the torques on the two axles is constant.

If there is no interaxle differential, then the two axles turn with the same speed. This condition translates in the model to the requirement that the slip of the two axles and the interface friction angle be the same. If the load on the two axles is different, the torque on the two axles computed on this condition is also different. The difference in the axle torques computed in the model corresponds to the phenomenon of torque windup.

The final step in the performance computations is the determination of the constants in the quadratic fitting the slip modified tractive effort curve. This information is transmitted through the subroutine arguments to the main program for further use.

#### USE OF THE MODEL FOR PREDICTION AND PARAMETRIC ANALYSIS OF VEHICLE PERFORMANCE

The vehicle performance model described previously incorporates many interactive features that have been generally recognized as having an effect on the performance of various off-road vehicles, yet have not been considered in other vehicle performance models, primarily because the general concept of these models was directed toward simplicity. One of the useful attributes of this vehicle performance model is its capability to analyze and assess the significance of these interactions under a wide variety of conditions and thereby gain further insight into the interrelationships that govern off-road vehicle performance. In the following discussion, results obtained by the vehicle performance model for a small number of selected combinations of the input variables are presented. These presentations are intended to show the value of the model as an analytical tool. The conclusions that may be drawn from the presented results are valid for that particular set of input data and should not be construed as generally valid. A systematic large scale analysis, covering a wide range of input conditions, is needed to draw general conclusions. Such an analysis is outside the scope of this work.

As mentioned earlier, the computer program for the vehicle performance model has been prepared so as to comply with the present structure of the AMC '74 Mobility Model. In this model the tractive force available in various gears at full throttle and the associated slip are determined and a tractive effort vs speed curve is obtained. In the following presentation the same relations obtained by the vehicle performance model are shown for various conditions, together with the theoretical tractive force that the engine

is capable of developing. All presentations refer to the M-151, 1/4 ton Army utility truck (commonly known as the "Jeep") that has the following vehicle characteristics.

Weight on front axle	1740 lbs
Weight on rear axle	1460 lbs
Wheel base	85 in.
Height of center of gravity (C. G.)	13 in.
Distance of C. G. from rear axle	46 in.
Tire diameter	30.8 in.
Tire width	7.15 in.
Tire section height	7.40 in.
Inflation pressure	15 psi
Deflection, front tires	1.31 in.
Deflection, rear tires	1.14 in.

Figure 3 shows the predicted four wheel performance of the M-151 1/4 ton truck in sand (average CI = 36), upslope (grade = 40%), level, and downslope (grade = -40%). The tractive force shown in the Figure does not include the tangential component of the vehicle weight, therefore, the net tractive force would be less when going upslope and more when going downslope. It is interesting to note that the tractive performance is, for all practical purposes, the same up, level, and downslope even though the weight distribution between the axles is different in each case. This appears to justify present methods of tractive force computations on slopes that disregard the effect of interaction due to the redistribution of axle weights on slopes. However, further systematic analysis is necessary to generalize this conclusion. An average cone index of 36 corresponds to a fairly compact sand where the wheel loads of the M-151 are not critical. It is possible, that under marginal trafficability conditions the axle weight redistribution could make the difference between "go" and "no go" conditions.

Another interesting feature of Fig. 3 is the decline of the tractive force with increasing torque in the first and second gears. This decline is consistent with the results of experiments and off-road driving experience. The application of excessive torque tends to spin out the tire, increase its sinkage, and reduce the tractive force that the tire can develop. In the present method of tractive effort calculation in the AMC '74 Mobility Model, this declination of the tractive force is suppressed to allow the application of the "full throttle concept" to the determination of the maximum speed that the vehicle can develop under the given conditions. Should the realistic tractive force variation with speed predicted by the vehicle performance model cause any problem with the subsequent

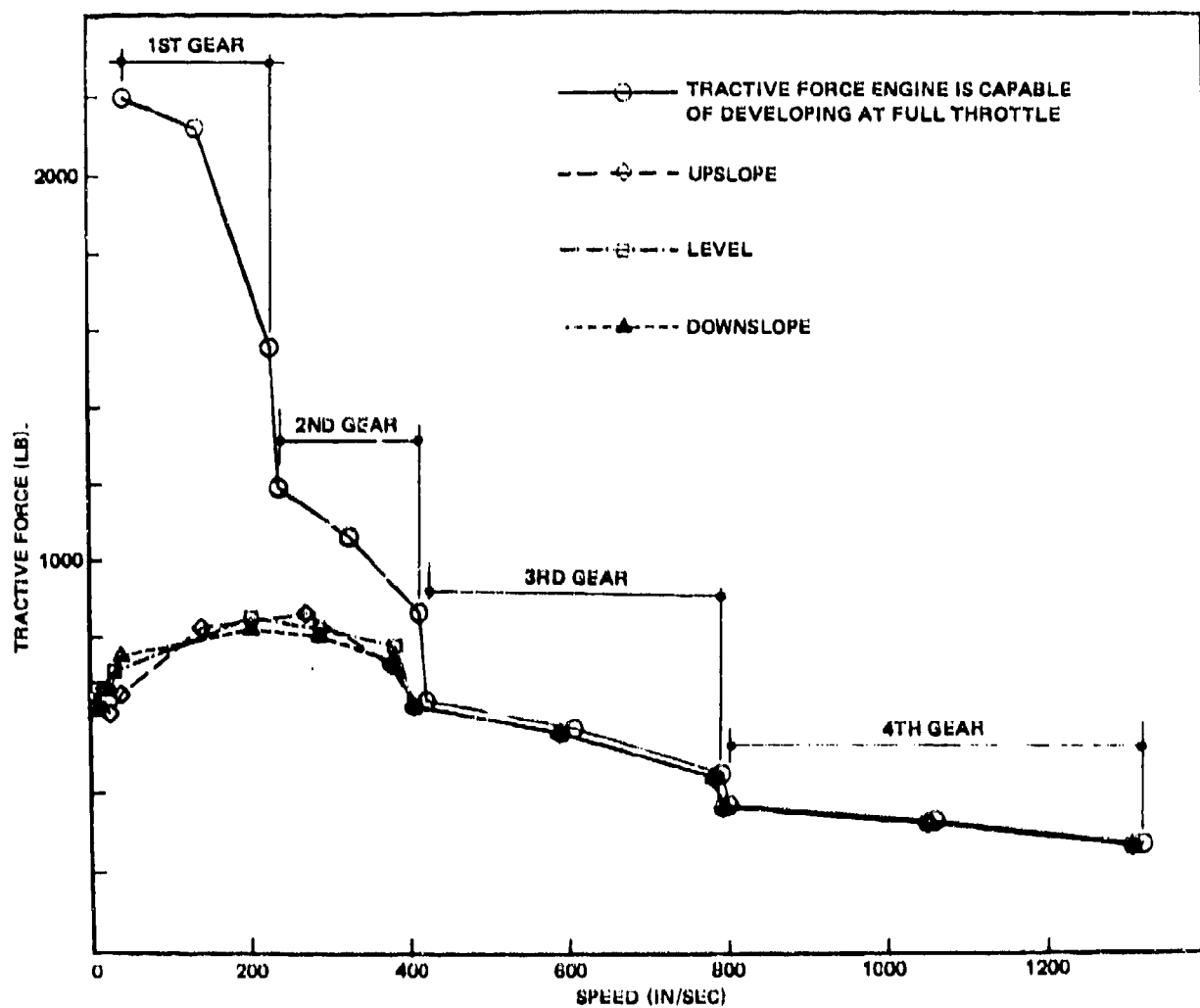


Fig. 3. Predicted Four Wheel Drive Tractive Performance of the M-151 1/4 ton Army Truck in Sand.

calculations in the AMC '74 Mobility Model, it may be necessary to assume that the tractive force is constant in that speed range where decline occurs.

The difference between the tractive force that the engine is capable of developing and the one actually developed by the vehicle is the motion resistance. It is seen that in contrast to the constant motion resistance concept adopted in the AMC '74 Mobility Model, the motion resistance strongly varies over the whole range of speed. However, Fig. 3 should not be interpreted as if the motion resistance generally decreased with speed; the motion resistance decreases because the applied torque decreases with speed in the "full throttle" concept. The low motion resistance at high speed is associated with the low applied torque in that speed range. The same torque applied at less than full throttle in lower gears would result in the same low motion resistance.

The interaction between applied torque and motion resistance is one of the most important processes that affect mobility. The vehicle performance model presented in this report simulates this interaction well, while the constant motion resistance concept ignores this important interaction completely. For a given vehicle the dependence of motion resistance on the applied torque could be systematically investigated by means of the vehicle performance model and approximate relationships between motion resistance and applied torque could be established. It is recommended that such an investigation be initiated and the results incorporated in the present structure of the AMC Mobility Model. A major weakness in the AMC Mobility Model thus would be eliminated and the structure and efficiency of the model preserved.

Figure 4 shows the tractive performance of the M-151  $\frac{1}{2}$  ton truck in sand with only the rear axle engaged. A comparison of this Figure with the previous one clearly shows the enormous advantage of the four wheel drive in sand.

Figure 5 shows the upslope (grade = 40%), level, and downslope (grade = -40%) tractive performance of the M-101  $\frac{1}{2}$  ton truck in clay (CI - 30), while in Fig. 6 the level ground tractive performance of the four wheel and rear wheel drive in clay is compared.

In Fig. 7 the upslope (grade = 40%) performance of the M-151  $\frac{1}{2}$  ton truck is compared with a similar vehicle equipped with interaxle differential. In sand, the interaxle differential would help traction development in the speed range where the tractive force declines with the decrease of speed (refer to Fig. 3). Analyses performed for other conditions indicate that the tractive performance with no interaxle differential is generally equal to or better than that with an interaxle differential. The magnitude of the torque windup that occurs, if there is no interaxle differential, can also be estimated by the

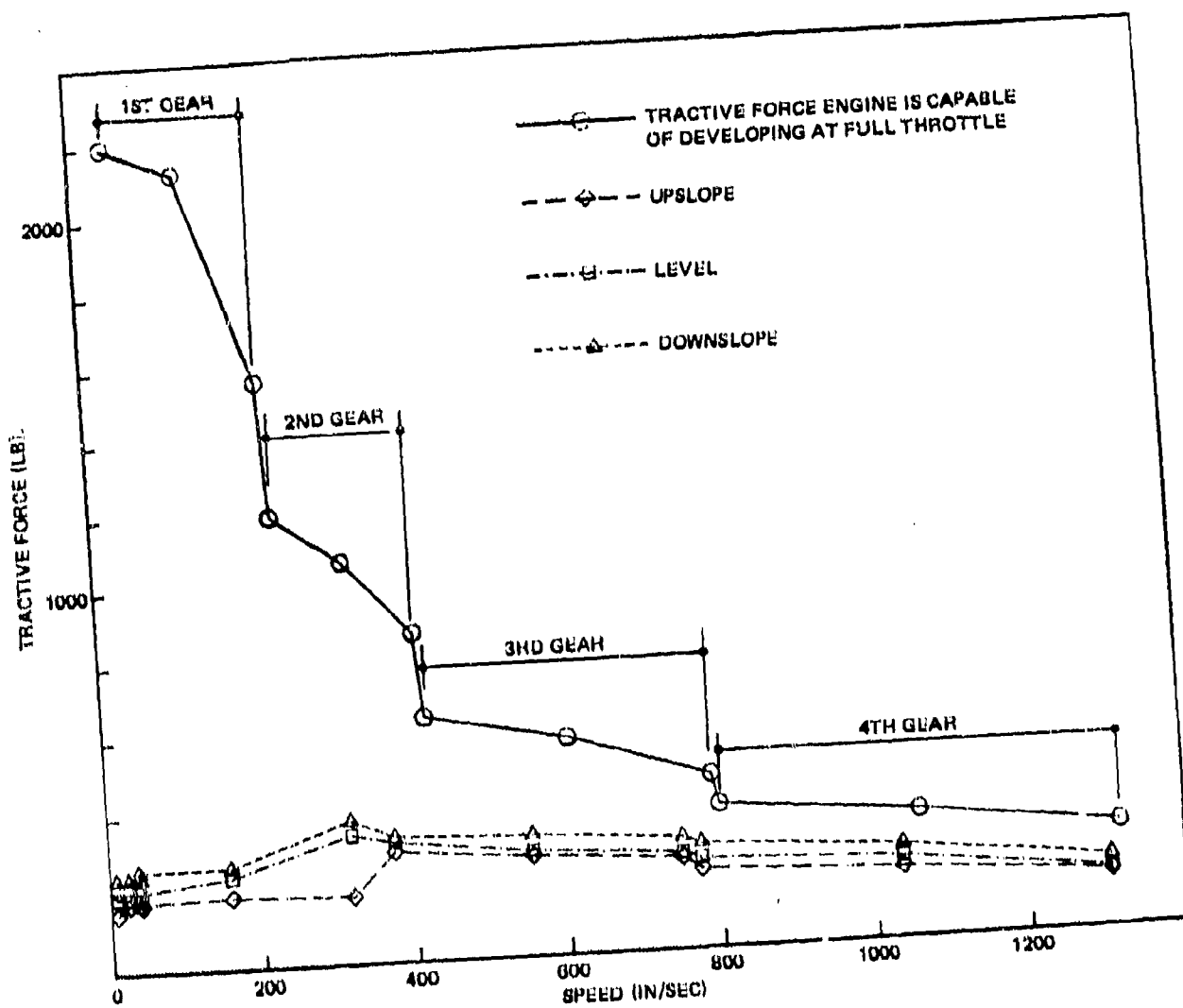


Fig. 4. Predicted Rear Wheel Drive Tractive Performance of the M-161 1/4 ton Army Truck in Sand.

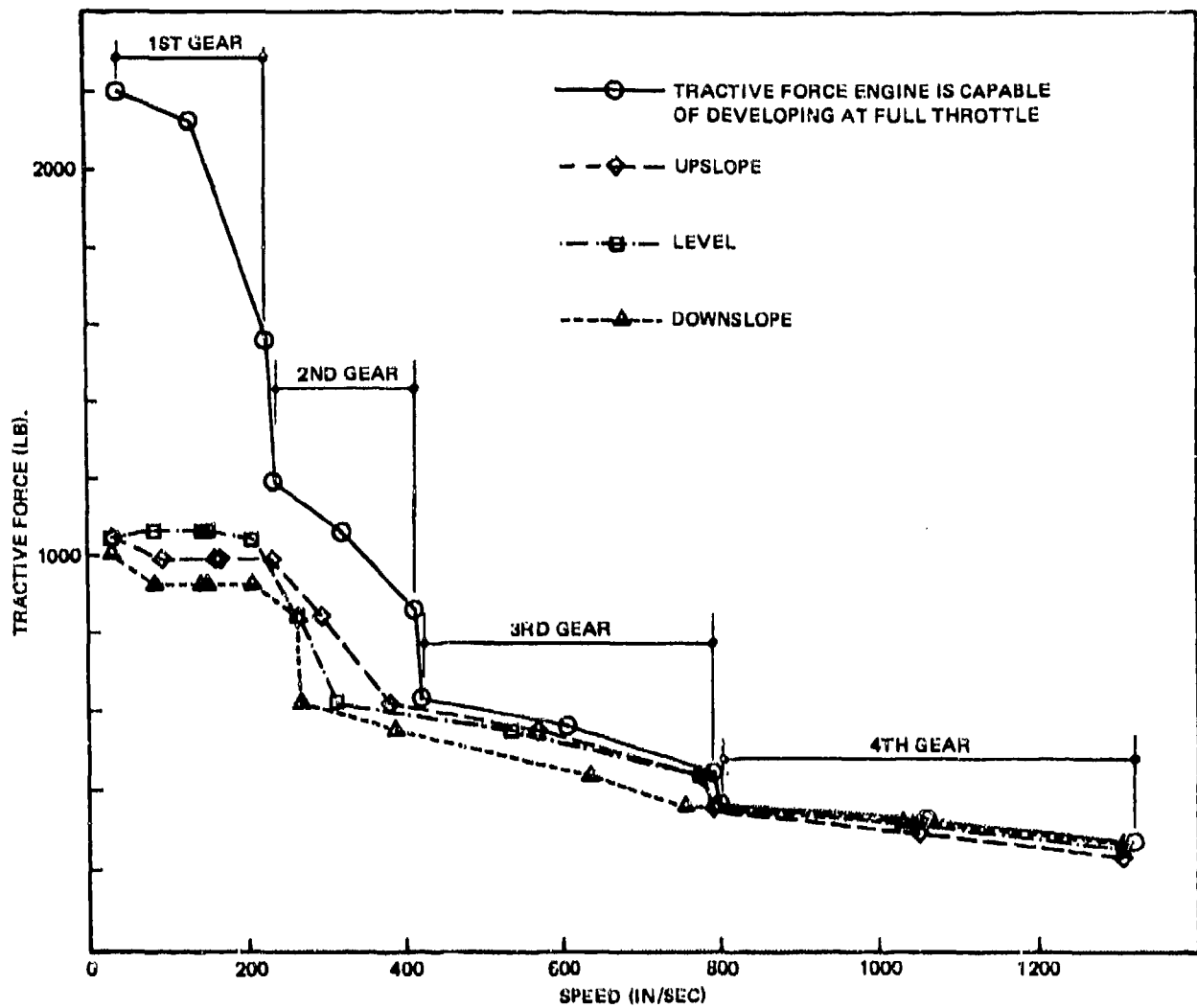


Fig. 5. Predicted Four Wheel Drive Tractive Performance Of the M-151 1/2 ton Truck in Clay (CI-36).



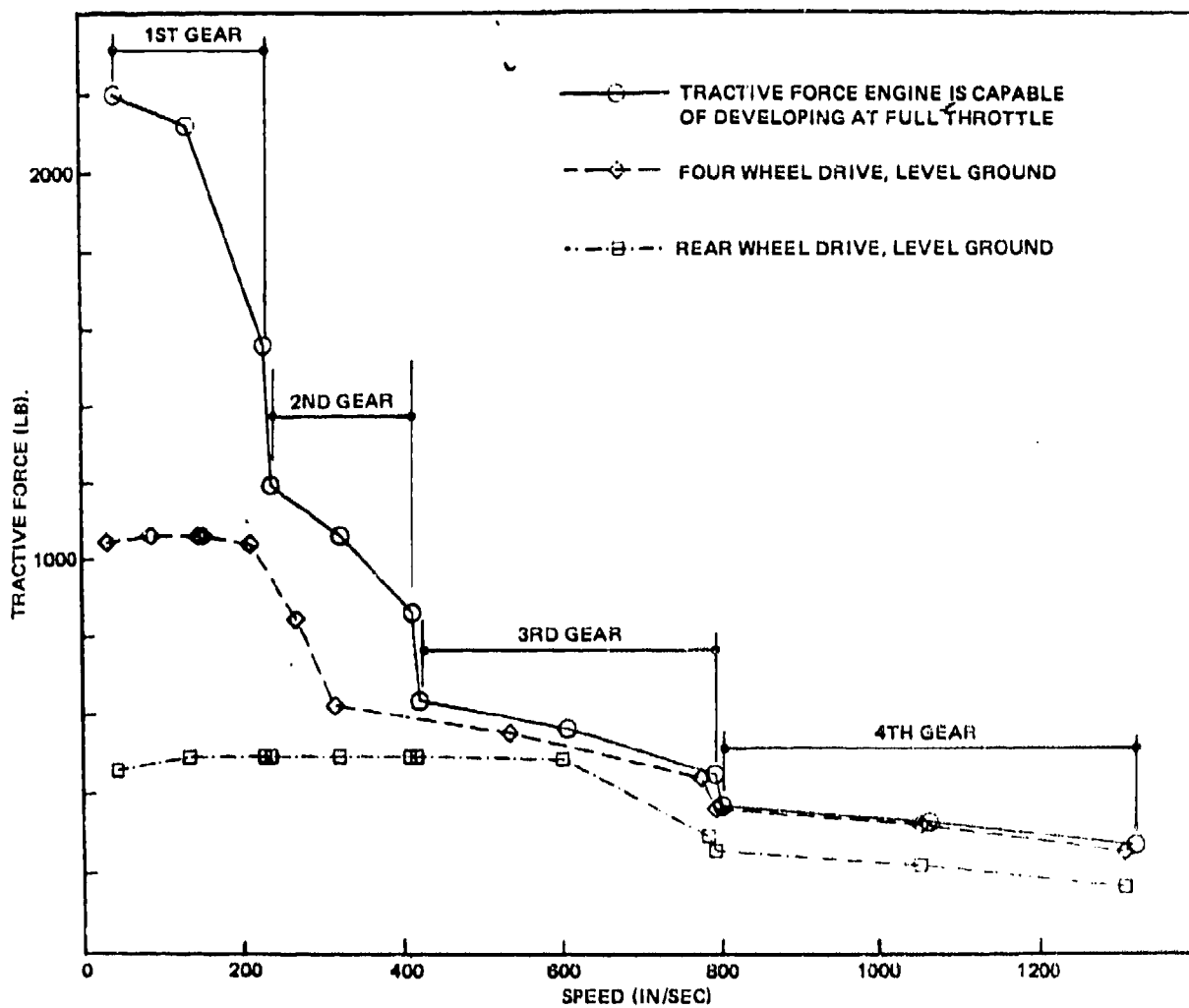


Fig. 6. Comparison of Four Wheel and Rear Wheel Drive Tractive Performance of the M-151 Truck in Clay.

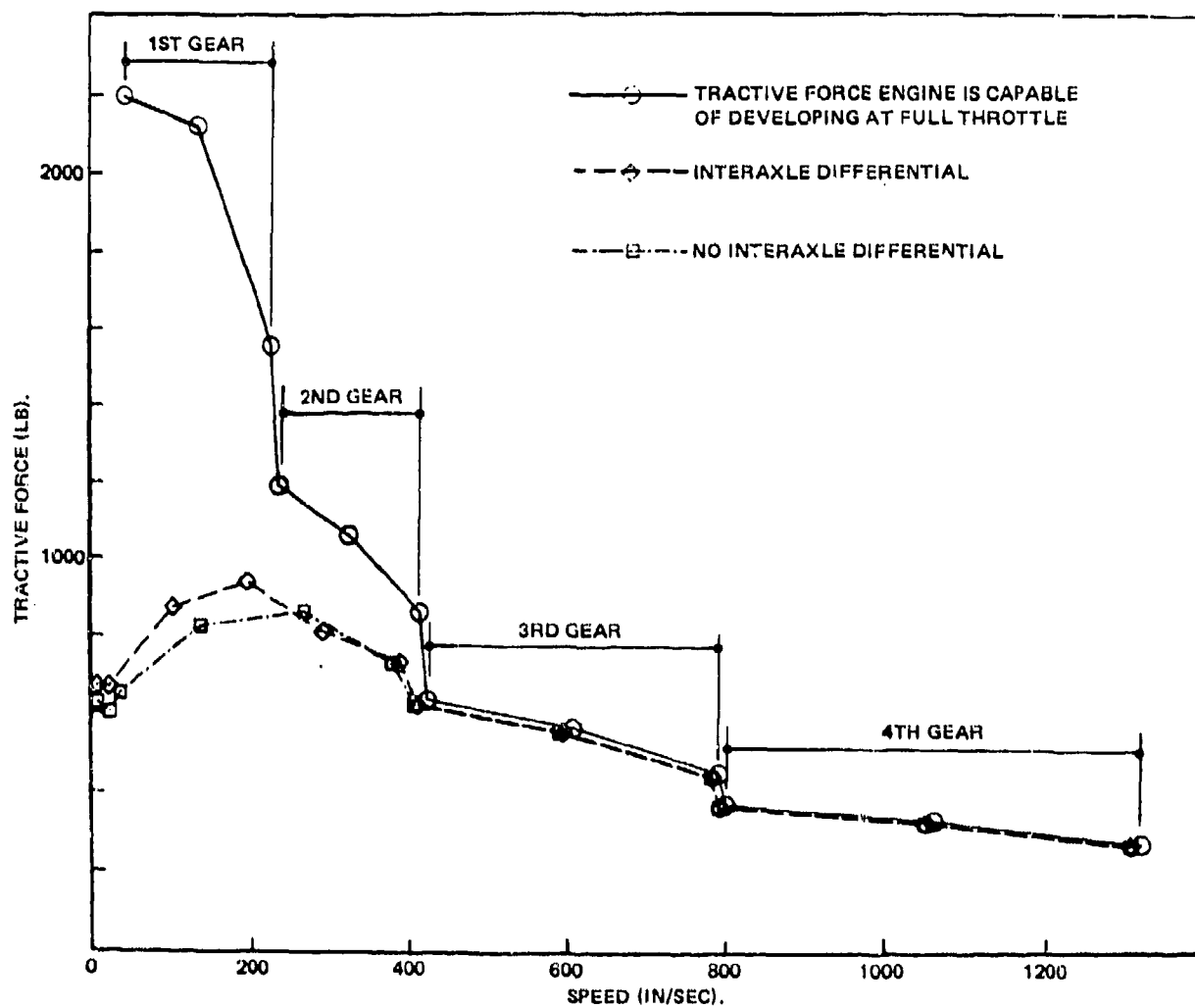


Fig. 7. Four Wheel Drive Tractive Performance in Sand With and Without Interaxle Differential.

vehicle performance model. Figure 8 shows the torque windup (in terms of axle torque) in sand up, level, and down slope. Another analysis indicated that in clay and level ground the sign of the torque differential changes as the gear is changed from 2nd to 3rd. Thus, in off-road travel where local slopes and gears are continuously changing there is no buildup of the differential torque, a fact that has been known from experience but for which no analytical explanation has ever been offered.

Figure 9 shows the effect of soil strength on tractive performance in sand. The Figure is a confirmation of the widely acknowledged notion that soil strength is the most important factor in off-road mobility. Finally, Fig. 10 shows an example of up (grade = 40%), level, and downslope (grade = -40%) tractive performance prediction by the vehicle performance model in cohesive-frictional soils ( $\phi = 25^\circ$ ,  $c = 100$  lbs/sq ft). The tractive performance on level ground shown in the Figure is sometimes lower than up or downslope. The explanation of this seemingly paradoxical prediction is that the actual normal axle load on a slope is less than the axle load on level ground, therefore, the sinkage is less and the drawbar pull higher (the tangential component of weight is not included in the tractive force shown). It is of interest to note that the combination of a small cohesion and a relatively low friction angle is more advantageous for the development of tractive effort than either a purely frictional (see Fig. 9) or purely cohesive soil with high strength.

These examples are but a few samples of tractive performance predictions for selected combinations of input variables. Other conditions, such as partial throttle performance, inflation pressure and tire size variations, etc. could be readily analyzed by the vehicle performance model. It is recommended that the 4 x 4 model be extended to multi-axle configurations and used, in addition to mission analyses, for general parametric analyses of the tractive performance of off-road vehicles.

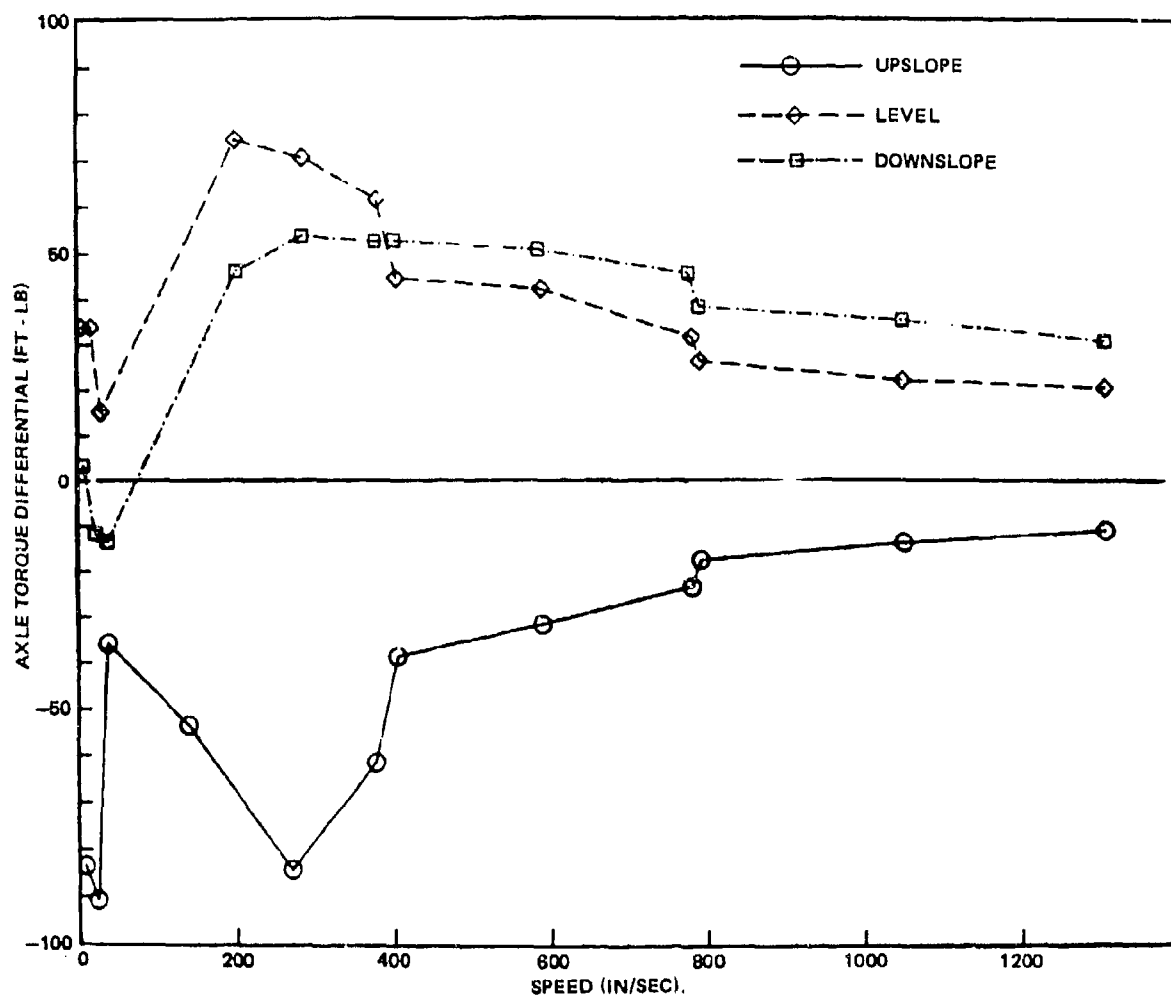


Fig. 8. Torque Differential Between Front and Rear Axle Developing in Sand (CI=36).

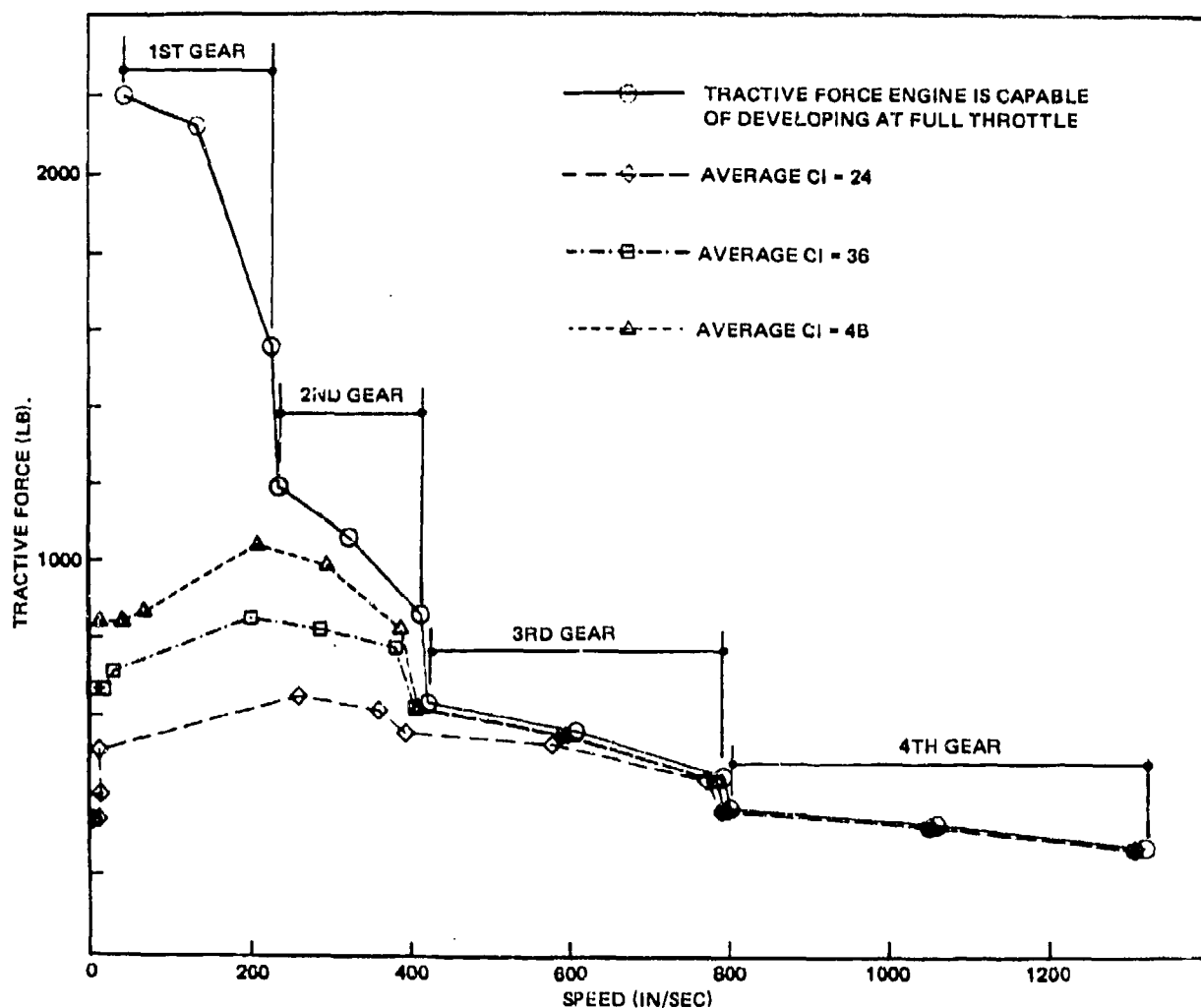


Fig. 9. Effect of Soil Strength on Tractive Performance in Sand, M-151 1/4 ton Truck.

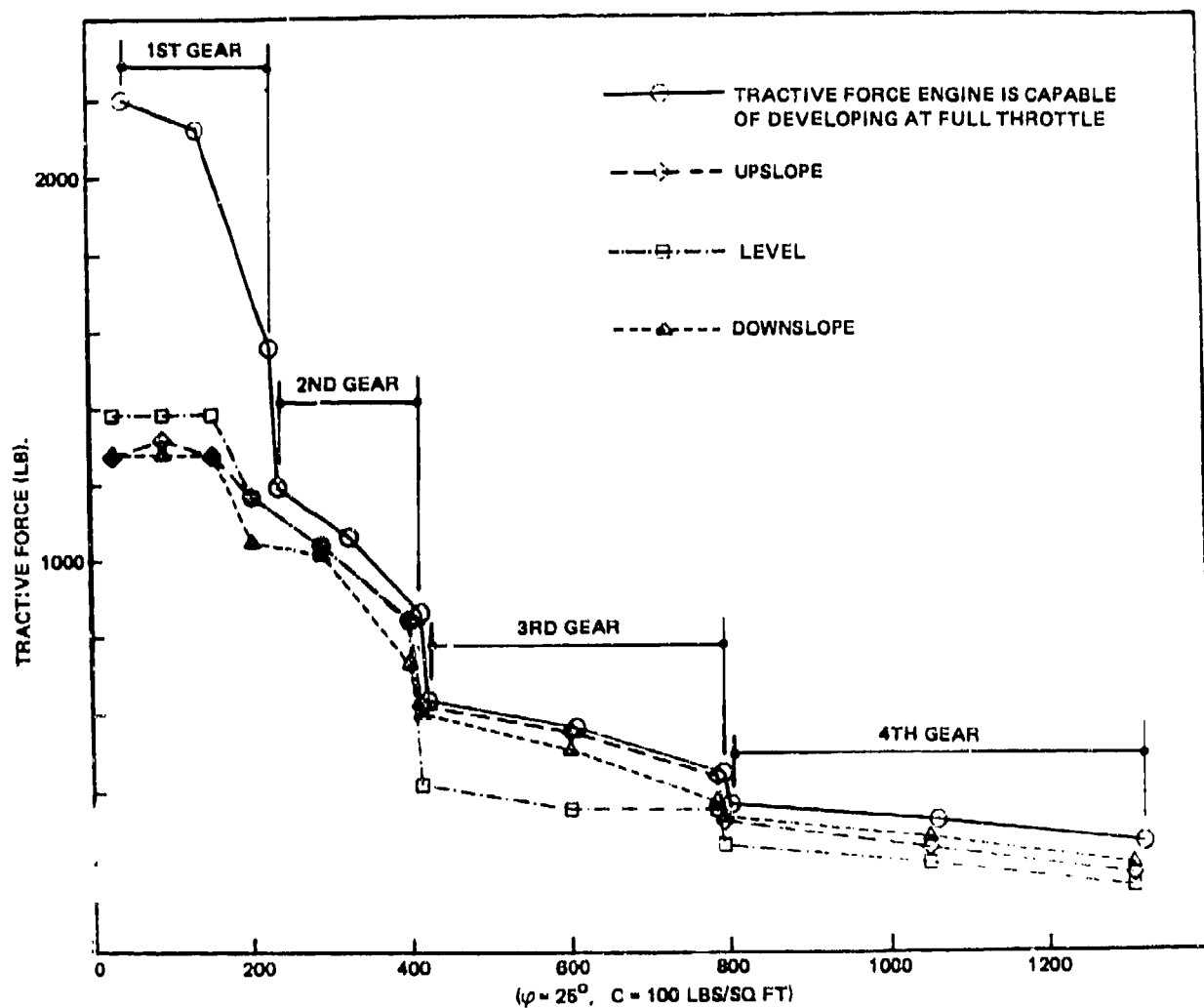


Fig. 10. Four Wheel Drive Tractive Performance of the M-151 1/4 ton Truck in Cohesive-Frictional Soil.

### 3. TIRE-SOIL INTERACTION MODEL FOR BRAKED CONDITIONS

#### CONCEPT

Braking exerts a negative axle torque that has to be balanced by soil reaction forces. The drag (negative drawbar pull) that results from the tire-soil interaction under the action of the negative torque is the total braking force that the tire can develop. The negative torque is balanced primarily by shear stresses at the tire-soil interface. Conceptually, tire-soil interaction under braked conditions is identical with that under driven conditions but for the sign of the interface shear stresses. The opposite direction of the shear stresses, nevertheless, results in certain conditions that have not been considered in the driven tire-soil interaction model. These are as follows.

a) In the driven tire-soil interaction model a "No Go" condition is indicated whenever the soil reactions were insufficient to carry the load, even if the entry and rear angles reached their maximum value. In the braked tire-soil interaction model this condition is identified with the development of the maximum braking effort that the braking system of the vehicle is capable of developing.

b) In the driven tire-soil interaction model provisions are made to account for situations when, under certain soil conditions, only one, rearward directed slip line field develops. This situation cannot occur under braked conditions but an analogous one develops when there is only one forward directed slip line field. However, the maximum value of the central angle that defines the position of the singular point is much larger for the forward directed single slip line field (it equals the maximum entry angle), than for the rearward directed one. The problems caused by this difference are discussed later.

c) In the driven tire-soil interaction model the conditions, when the soil is not stressed to the limiting plastic state, are identified as "HARD SURFACE CONDITIONS." These conditions, of course, are not critical for tractive performance but may be critical for braking performance. In the AMC '74 Mobility Model a braking coefficient (XBRCOF) obtained on pavement is assigned to this condition. This braking coefficient may not always materialize on hard soil. In the braked tire-soil interaction model a braking coefficient for hard surface conditions is defined as the ratio of normal stress (assumed to be equal to the limit pressure) to the shear strength of soil under that normal stress.

## DESCRIPTION OF BRAKED TIRE-SOIL INTERACTION MODEL

The computer program for the braked tire-soil interaction model is written as subroutine "BRAKE" to be called from the AMC '74 Mobility Model as an alternate to the total braking force submodel by the following arguments.

### INPUT VALUES:

GCW	=	Gross combination weight
GCWB	=	Gross combined weight on braked axles
GCWNB	=	Gross combined weight on non-braked axles
WGHT	=	See INPUT designations (for Vehicle Performance Model, Pg 6)
DLAW		"
SECTW		"
TPSI		"
DFLCT		"
SECTH		"
IST		"
RCIC		"
GRADE		"
CGR		"
CGH		"
TL		"
ISEAS		"
COHES		"
PHI		"
GAMMA		"
IP		"
IB		"
NTRAV		"
XBRCOF	=	Maximum combination braking coefficient
XBR	=	Maximum braking effort vehicle can develop



## OUTPUT VALUES:

TBF(j)	=	Total soil/slope/vehicle derived braking force up (j = 1), level (j = 2) and down (j = 3) slope
BFGONO	=	1 if vehicle braking is inadequate for down slope operation 0 otherwise

The computation scheme for the determination of braking force is similar to that described in Ref. 2 for the tire-soil interaction model for driven tires. Therefore, details of the computations that are identical with those described in Ref. 2 are omitted here and only the fundamental organization of the computations is discussed in the following description of the interaction model for braked tires.

The flow diagram showing the major steps in the computations is shown in Fig. 11. First, the Coulomb strength parameters are computed from the cone index values, if they are not given as input values. This computation is essentially the same as in the vehicle performance model. A "Do Loop" is entered for the computation of the braking force up, level, and down slope (if required) and for each axle. The weight on each axle is computed taking into account the redistribution of weight due to the slope and applied braking torque. On the basis of the maximum braking torque that the brakes can supply an estimate is made of the interface friction angle  $\delta$ . The slip line field computation routine (essentially the same as in the driven tire model) is entered and iteration is performed on the entry angle,  $\alpha_0$ , until the normal stress at  $\alpha_d$  (Fig. 12) matches the limit pressure,  $p_l$ . Then the rear field is computed and angle  $\alpha'_d$ , at which the normal stress matches the limit pressure, is determined. If the normal stress  $q_r$  at  $\alpha_r$  for an infinitesimal slip line field is higher than the limit pressure, then there is only one forward slip line field and the computations are repeated for a slip line field extending from  $\alpha_0$  to  $\alpha_r$ . The interface stresses obtained from the slip line field computations are integrated and the load, drawbar pull, and torque determined. Iterations on  $\alpha_r$  and  $\alpha_d$  are performed until the computed load agrees with the input load within the allowed tolerances. If the load computed for the maximum values of  $\alpha_0$  and  $\alpha_r$  is less than the input load, a "no go" situation exists and the available braking force is equated with the maximum that the vehicle can develop. Otherwise the available braking force equals the tangential force component obtained from the integration of interface stresses.

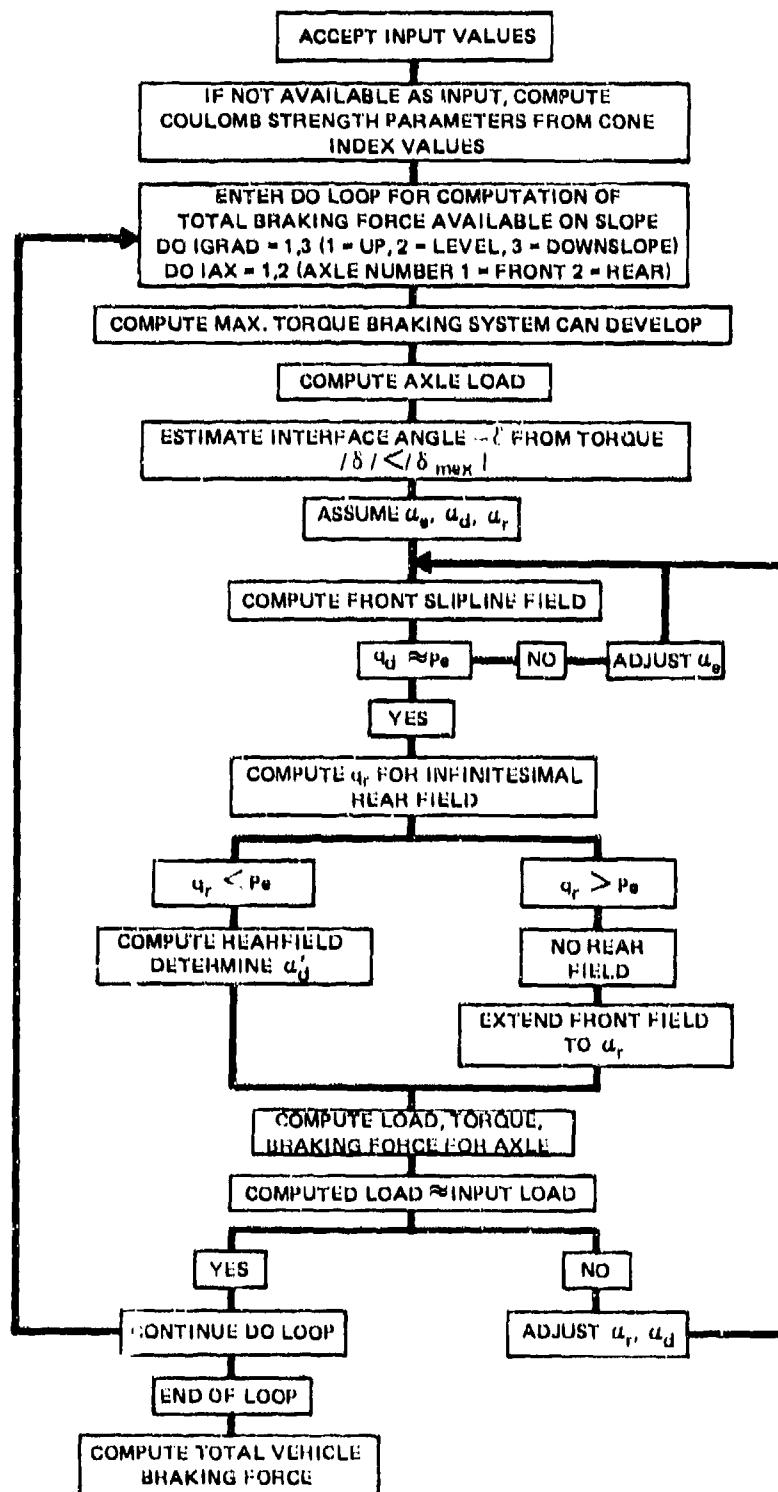


Fig. 11. Flow Diagram for the Computation of Total Vehicle Braking Force  
(For Designations see Fig. 12).

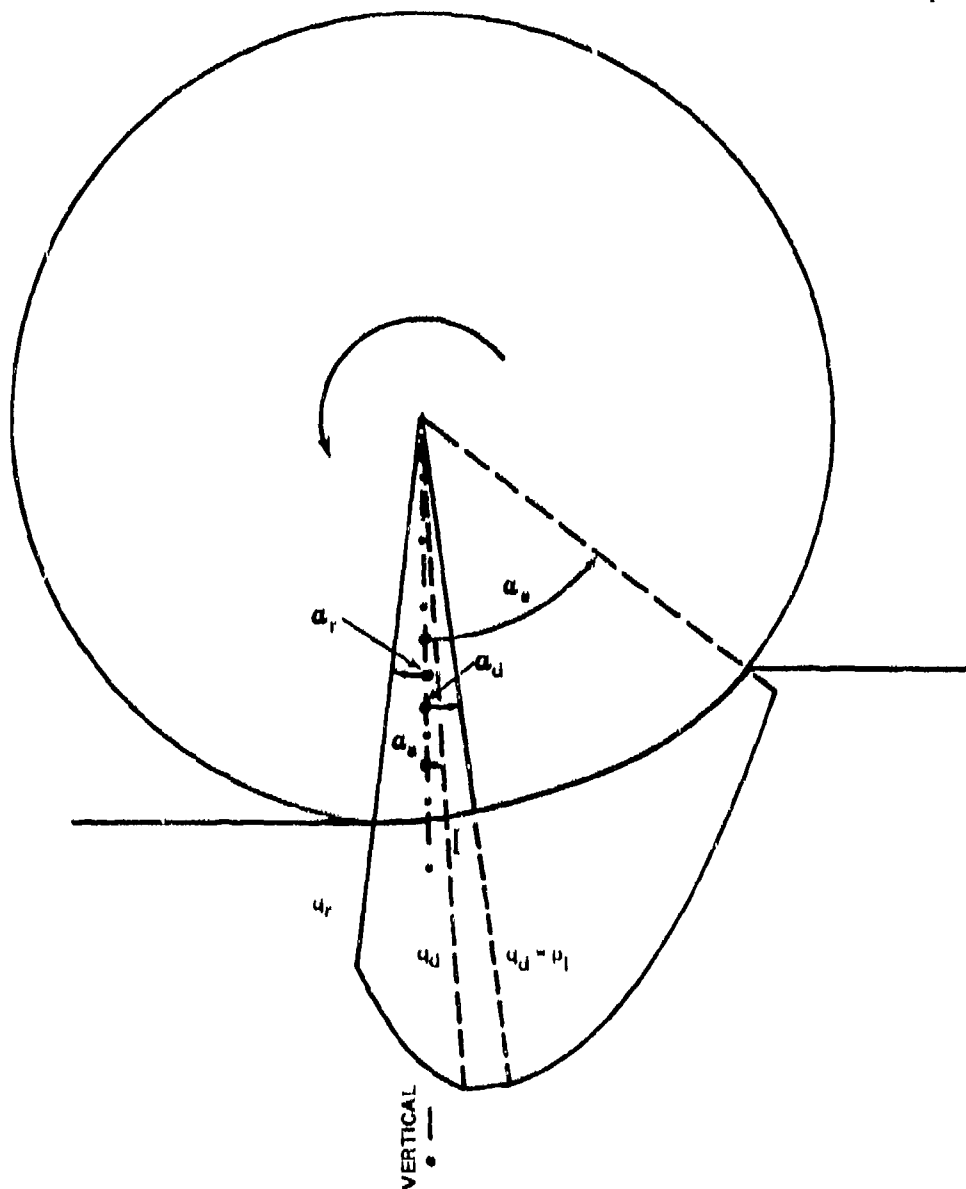


Fig. 12. Central Angles and Interface Normal Stresses in Braked Tire-Soil Interaction.

## PROBLEMS ENCOUNTERED

Unexpected problems arose with the iteration on  $\alpha_0$  aimed at determining the entry angle from the requirement that  $q_d$  match the limit pressure. This iteration scheme was designed in the driven tire-soil interaction model on the assumption that  $q_d$  monotonically increases with the entry angle  $\alpha_0$ . It was found that under certain braking and soil conditions this assumption is not valid. A computational routine was added to the program that recognizes this situation and directs the program to go to the sequences devised for the condition that for  $\alpha_0 = \alpha_0 \text{ max}$ , the normal stress  $q_d$  at  $\alpha_d$  is less than the limit pressure  $p_l$ .

Another problem was encountered in situations where conditions called for a single forward directed slip line field. Although this situation is analogous to the single rear slip line field condition occurring with driven tires, the maximum value of the central angle to the singular point in the slip line field is much higher in braked than in driven conditions. As a consequence, the forward directed single slip line field extends over a longer arc length and, if it is shallow, the "j" lines intersect the interface at a very acute angle. Under these conditions the determination of the locations of the nodal points at the interface becomes inaccurate and certain iterations become divergent.

Three methods were considered to remedy this situation:

- a) Use of a finer grid that would make the computations more accurate
- b) Local refinement of the computations
- c) Local reduction of the interface friction angle for the purposes of computation

For practical reasons method c) was adopted. It was found that a minor reduction of the local interface friction angle augments the angle of the intersection of the interface and slip lines sufficiently to eliminate this computational problem. Method a), although simple, would have required a significantly increased core space for the computations which was not readily available on the HP-3000 computer. Method a), however, may be considered when the program is used with the CDC 6600 computer available for the Mobility Model. Method b) would have required elaborate programming not justified by the significance of the problem.

#### 4. USE OF COULOMB STRENGTH PARAMETERS OF SOIL AND CONE INDEX VALUES IN MOBILITY EVALUATION

##### INTRODUCTION

In the vehicle performance model the soil is modeled by its Coulomb strength parameters that, for the purpose of mobility evaluation, are suitable for the characterization of practically every type of soil. The use of these fundamental parameters allows the application of soil mechanics theories and, specifically, the plasticity theory to the problems of mobility.

On the other hand, for the field determination of soil properties, cone penetration tests are extensively used. Dimensional analyses indicated that if the soil is either purely frictional or purely cohesive then cone index values are sufficient for the characterization of soils and simple relationships between dimensionless tire performance parameters and so-called "numbers" can be established experimentally. The experimental information available today on tire performance in these two types of soil, that represent extremes of the soil spectrum, is invaluable for mobility evaluation.

One reason behind centering the experimental research around these two extreme types of soils was that these are also the soils where "no go" situations most frequently occur. Today, when agility as well as general mobility is required of combat and support vehicles, critical situations for agility are more likely to occur in the general class of frictional-cohesive soils than in the extremes of the soil spectrum. Thus, it is increasingly important to characterize soils by their fundamental Coulomb strength parameters rather than cone index values.

##### RELATIONSHIPS BETWEEN CONE INDEX VALUES AND COULOMB STRENGTH PARAMETERS

In the AMC '74 Mobility Model the terrain is characterized by a single cone index value that represents the average cone index in the upper six inches of soil. Obviously, a single value cannot be uniquely related to several independent parameters and relationships among them can only be developed if one of the Coulomb parameters is nearly zero, as in the case of either purely frictional or purely cohesive soils. In these cases the following approximate relationships have been developed earlier (Ref. 2).

## Frictional Soils

A relationship between friction angle and cone index gradient has been established indirectly by using empirical relationships between relative density and friction angle and estimating the frictional angle on the basis of relative density.

The relative density of frictional soils may be expressed as (Ref. 2)

$$D_r = 71.1 \log G + 11.33 \pm 6.8 \quad (1)$$

where  $D_r$  = relative density (%)  
 $G$  = cone index gradient (psi/in.)

The friction angle ( $\phi$ ) is estimated from the following relationship

$$\cot \phi = 1.64 - 0.68 * D_r \quad (2)$$

## Cohesive Soils

The following empirical relationship is used to estimate the Coulomb strength parameters for cohesive soils (Ref. 2).

$$\begin{aligned} C \text{ (psi)} &= CI/12.5 \\ \phi \text{ (degrees)} &= CI/4 \end{aligned} \quad (3)$$

The friction angle of purely cohesive soils is theoretically zero. However, real soils that fall in the category of cohesive soils exhibit a small friction angle as indicated by Eq. 3. The use of a small friction angle is also advantageous from the computational point of view. A number of computational schemes are not applicable for a  $\phi = 0$  condition and the additional algorithms needed to provide for this contingency would increase the length of the program appreciably.

## APPLICATION OF PLASTICITY THEORY TO THE CONE PENETRATION PROBLEM

Plasticity theory may be applied to the determination of cone penetration resistance in soils that exhibit relatively small volume change necessary to develop their shear strength. In such soils the cone index ( $CI$  = cone penetration resistance/base area) may be expressed as

$$CI = f(c, \phi, \gamma, \delta, t) \quad (4)$$

where  $c$  = cohesion

$\phi$  = friction angle

$\gamma$  = unit weight of soil

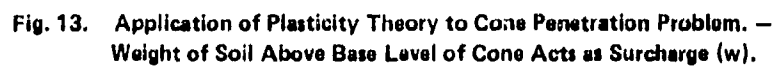
$\delta$  = interface friction angle

$t$  = depth of base level beneath the surface

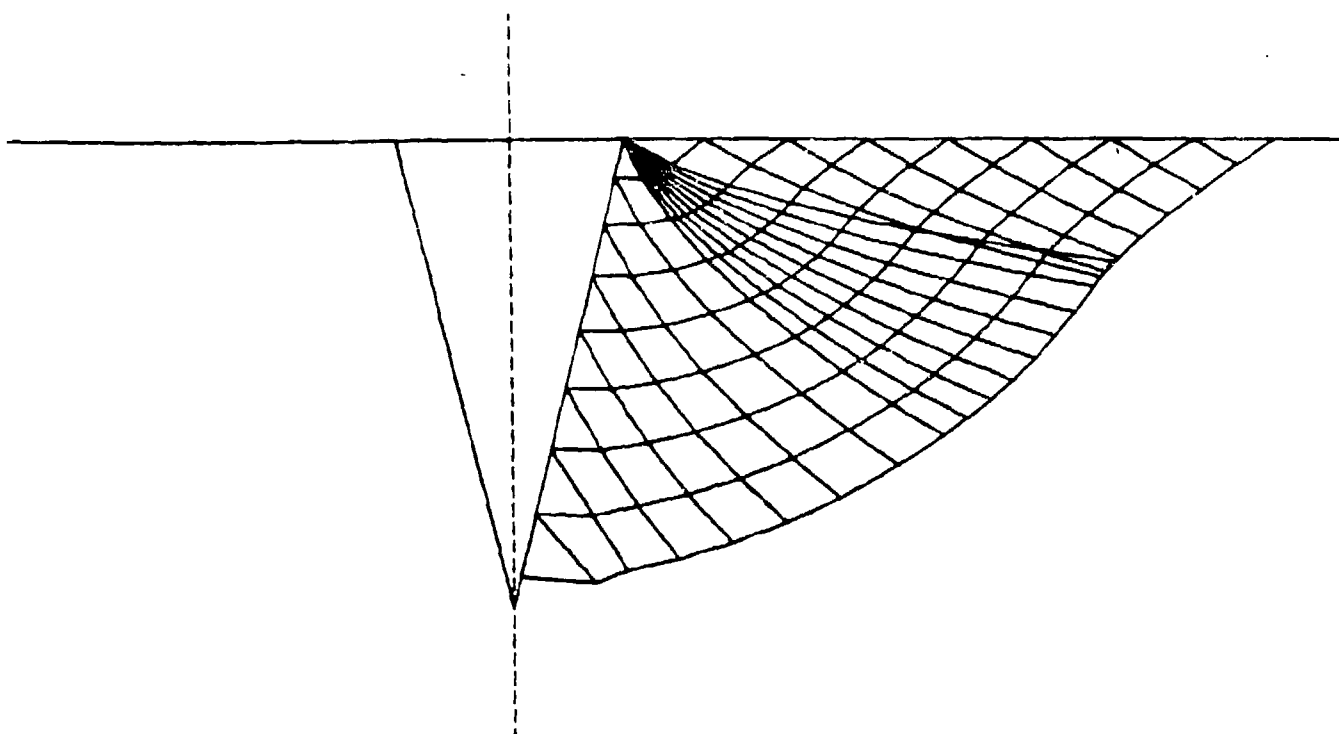
In the field determination of soil properties the unit weight of soil ( $\gamma$ ) is rarely measured. If estimated values of  $\gamma$  are used a source of inaccuracy is introduced in the above relationship. Another source of inaccuracy is the value of interface friction angle  $\delta$ , that depends on the friction developed at the cone face. Very little is known about the actual values of  $\delta$  that are likely to vary with soil conditions and the roughness of the cone face. The uncertainties in the value of  $\gamma$  and  $\delta$  set a certain limitation to the accuracy of any relationship that can be developed on a theoretical basis between the Coulomb strength parameters and cone index values. It is for this reason that no attempt was made to refine the approximate relationships (Eqs. 1, 2, and 3) established empirically.

The cone index value also depends on the depth of base level beneath the surface. If an average CI is given this depth is assumed as 3 inches. In an earlier Grumman research project plasticity theory methods were applied to the problem of cone penetration (Ref. 4). An interactive computer program was developed in this program for the computation of cone penetration resistance on the assumption that the soil above the base of the cone acts as a surcharge (by its weight only) as illustrated in Fig. 13. The slip line field shown in the Figure is the solution of the differential equations of plasticity for the axially symmetric case obtained by numerical solution methods. This research project revealed that for certain cone angles and  $c - \phi$  values it was not possible to close the slip line field at the apex, and for some conditions, overlapping of the slip line field occurred (Fig. 14a). Such overlapping is physically not permissible since two different stress states cannot exist in the soil at the same location and same time.

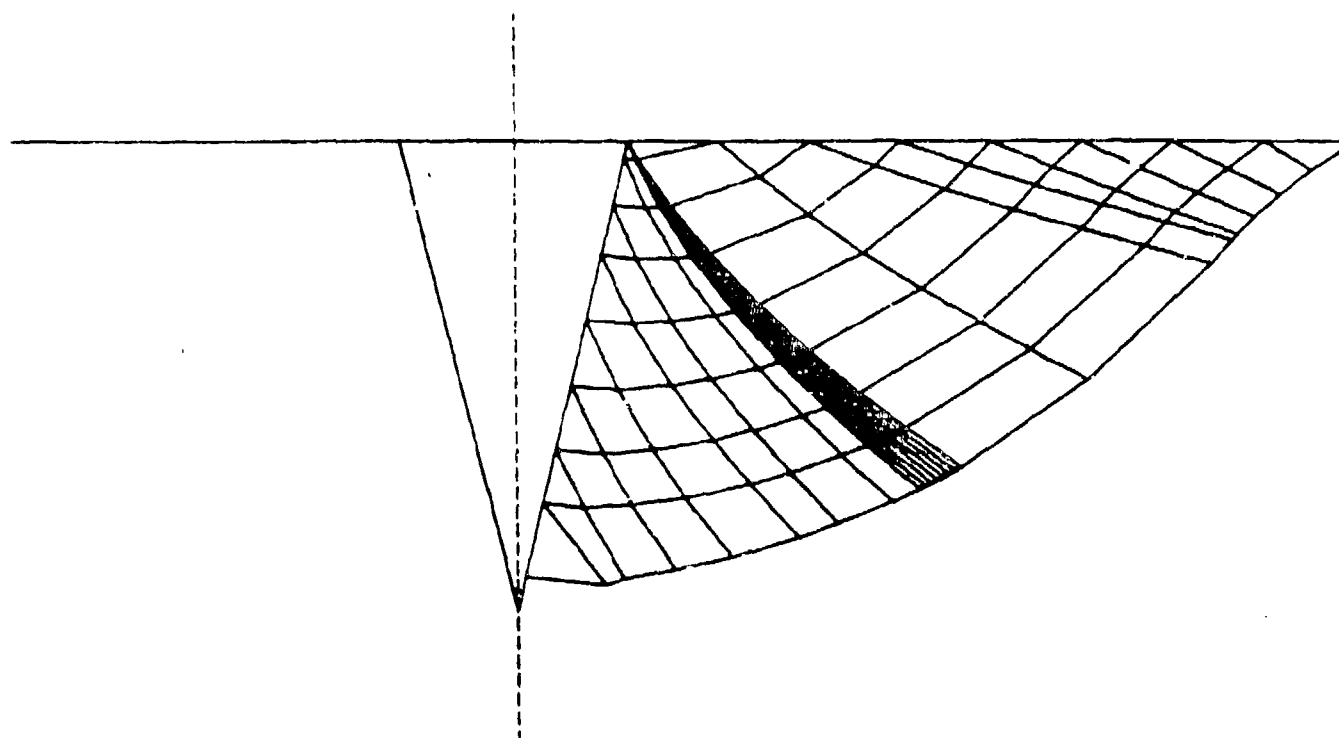
To resolve these problems further theoretical studies were made for the present project. These indicated that for the  $30^\circ$  apex angle WES cone axial symmetry requires a different direction of the major principal stress at the apex than that specified as boundary condition for the face of the cone, including the apex, as the direction corresponding to the assumed interface friction angle. Thus, in the immediate vicinity of the apex plastic state and axial symmetry pose conflicting requirements and, therefore, some other than a plastic state (probably rigid) must exist there. Fortunately, for the total cone resistance this problem has little significance since the area involved is very small. In the following analyses 1/100 of the base area (within one tenth of the base radius) was excluded from the slip line field computations to solve the problem. It is believed that the exclusion of such a small area does not affect the results appreciably.







a) OVERLAP IF SOIL ABOVE BASE LEVEL OF CONE IS TREATED AS SURCHARGE



b) NO OVERLAP WITH INCREMENTAL PENETRATION METHOD

Fig. 14. Slip Line Fields for Cone Penetration Problem.

The overlapping problem was resolved by treating the depth effect differently than in the earlier project. Instead of assuming that the soil above the base of the cone acts by its weight only, it is assumed that the stress state in the soil caused by the penetration of the cone is "locked in." The penetration problem is solved in an incremental way: starting from the surface, the stress state at a depth  $\Delta z$  is determined from the slip line field and in the next step the slip line field is computed for the condition that the base of the cone is at depth  $\Delta z$  and the boundary conditions at that level outside the base area (previously assumed as surcharge corresponding to  $\Delta z$  depth) are determined from the "locked in" stress state. The stress state at this depth has been determined in the previous slip line field computation. When the boundary conditions correspond to this "locked in" stress state, there is no overlapping (Fig. 14b). Variation of cone penetration resistance with depth computed by this method is shown in Fig. 15 for three values of the cohesion.

This method is suitable for determination of the variation of cone penetration resistance with depth, if the Coulomb strength parameters, the unit weight ( $\gamma$ ), and the interface friction angle ( $\phi$ ) are known. It is interesting to note that even for a  $\phi = 20^\circ$  material, the cone penetration resistance remains constant beneath a certain depth. This explains the often observed phenomenon that cone index signatures in frictional-cohesive soils often resemble those obtained in purely cohesive soils.

This method may be used to establish Coulomb strength parameters from cone index profiles by trial and error. Further theoretical and experimental research (including interface friction angle measurements) is needed to develop a procedure that uniquely converts cone index profiles to Coulomb strength parameters. It is strongly recommended that further field cone penetration tests be conducted in such a way that the complete depth profile of cone resistance be available for possible conversion to Coulomb strength parameters.

#### EFFECT OF THE PASSAGE OF LEAD WHEEL ON THE PROPERTIES OF SOIL ENCOUNTERED BY TRAILING WHEEL

##### Frictional Soils

The passage of a tire generally increases the relative density. This increase in relative density depends on the limit pressure. The estimated relative density after passage of the lead wheel is

$$D'_r = P_l (1 - D_r) / 50 \quad (5)$$

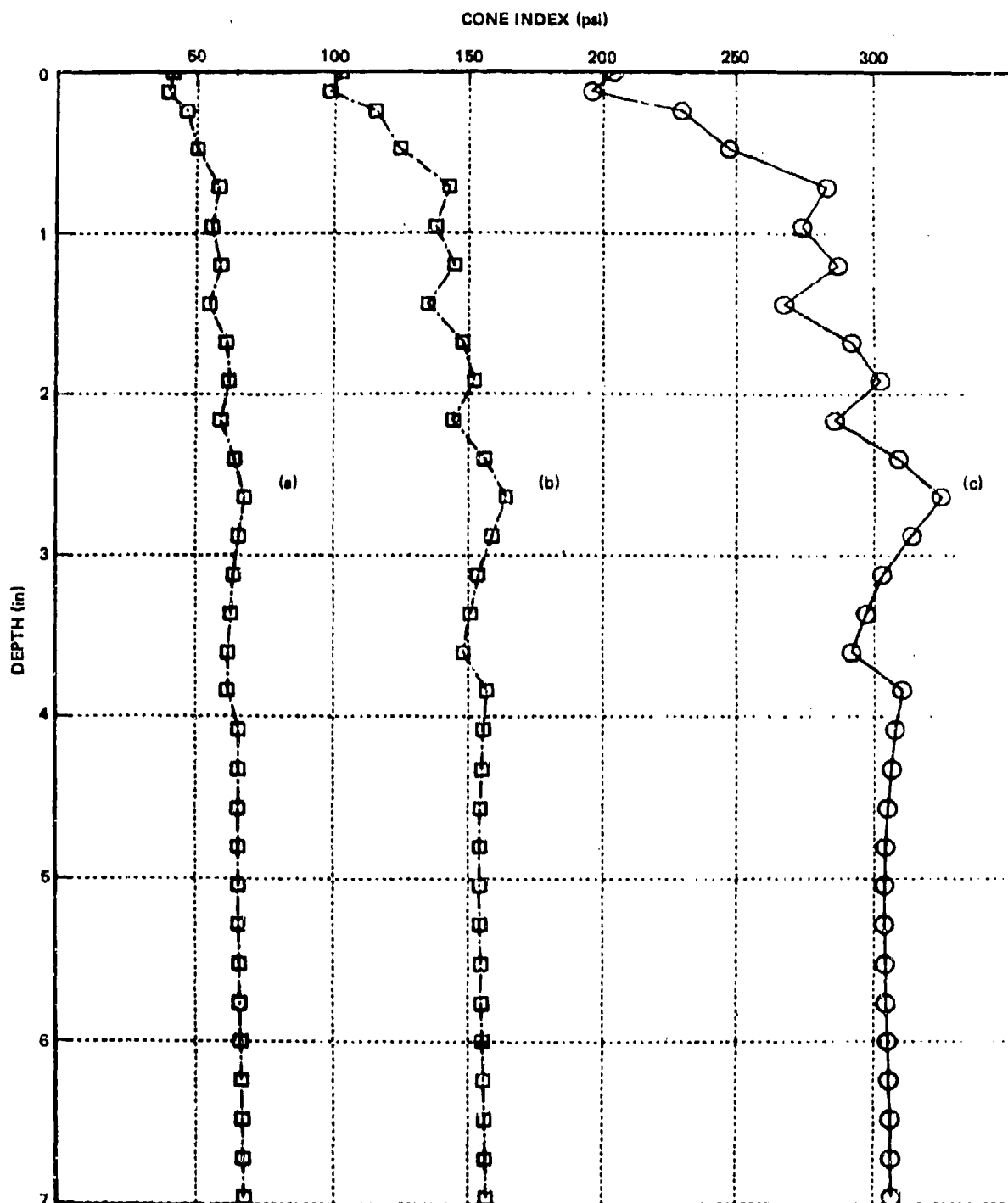


Fig. 15. Cone Penetration Profiles Determined by the Incremental Penetration Method. Friction Angle  $\phi = 20^\circ$ , Cohesion a) 200, b) 500, c) 1000 lbs/sq ft.

where  $D_r$  = initial relative density  
 $D'_r$  = relative density after passage of lead wheel  
 $P_l$  = limit pressure for lead wheel

The maximum value of  $D_r$  computed by Eq. (5) is set at  $D_r = 1.1$  ( $D_r = 1$  is the maximum relative density obtainable by laboratory procedures. In the field somewhat higher densities may be obtained).

#### Cohesive Soils

Soils in this category are close to 100% saturation at which preloading by a lead tire does not increase the soil strength. Multiple passage of vehicles may even destroy some of the structural strength of this type of soil.

#### Cohesive-Frictional (c- $\phi$ ) Soils

Cohesive-frictional soils encountered at the surface are generally not saturated. The presence of air voids in these soils allows the soil to compact under the stresses applied by the lead wheel to the soil. If the degree of saturation is less than, say, about 85% then it may be assumed that the stresses in the soil are "effective stresses", i.e., no part of the applied stresses is carried by pore water or air pressure. Once the soil acquired some strength in its effective stress history, a major portion of this strength remains "locked in" even after the stresses are released. This concept may be used for the estimation of the increase in soil strength due to the action of the loading wheel (Fig. 16). The maximum normal stress that the soil experiences beneath the lead wheel determines the strength that remains partially locked in the soil. The degree of "locking in" is expressed by the coefficient  $K_g$  that is applied to the friction angle in the stress range up to the estimated max. normal stress.  $K_g$  may be determined by triaxial tests that duplicate the stress path in the soil during passage of the lead wheel.

Another way of obtaining information on the effect of compaction on the strength properties of soil would be to make cone penetration tests in the rut of vehicles. In the field there is generally some off-road vehicle that carries the crew. It would require little additional work, to make cone penetration tests in the rut of the vehicle and record it together with the information on the vehicle tire characteristics.

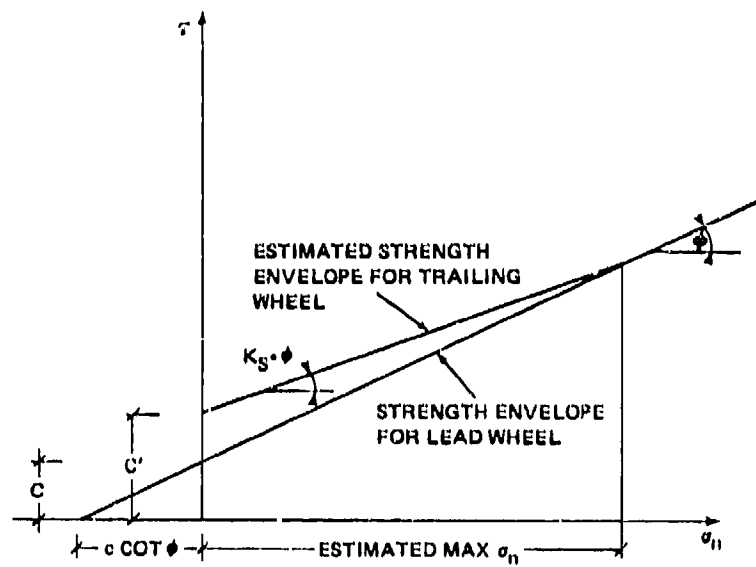


Fig. 16. Estimation of the Effect of Compaction by the Lead Wheel on the Strength Properties of Soil Encountered by the Trailing Wheel.

## 5. CONCLUSIONS AND RECOMMENDATIONS

A vehicle performance model has been developed that incorporates all essential interactive features of vehicle soil interaction that affect vehicle performance. The model simulates vehicle performance with sufficient accuracy for both performance prediction and parametric analyses of conceptual and existing vehicles. A braked tire-soil interaction model has also been developed to complement the vehicle performance model.

It is recommended that the interactive capability of the vehicle performance model be used to establish applied torque-motion resistance relationships for the various military vehicles. The use of such relationships in the AMC '74 Mobility Model would enhance the performance simulation therein without significant changes in the present structure of the model. It is also recommended that the present 4 x 4 vehicle performance model be expanded to multi-axle configurations.

In the soil categories presently used in the AMC '74 Mobility Model frictional-cohesive soils are not included. It is recommended that a new soil category comprising these soils be included in the Model and the vehicle performance model be used for the prediction of tractive performance in this soil category. For the characterization of soils in this category it is necessary to establish their Coulomb strength parameters. For this purpose it is recommended that

- a) Complete penetration resistance-depth profiles be obtained in future field investigations, both on virgin soil and after the passage of a specified vehicle
- b) Further research, based on the new method of analysis of continuous cone penetration (reported in Section 4), be conducted to develop methods for the direct conversion of resistance profiles to Coulomb strength parameters.

## 6. REFERENCES

- 1) Jurkat, M.P., Nuttall, C.N. and P.W. Haley, "The AMC '74 Mobility Model," U.S. Army TACOM Mobility Systems Laboratory Technical Report No. 11921 (LL-149) May 1975.
- 2) Karafiath, L.L., "Development of Mathematical Model for Pneumatic Tire-Soil Interaction," U.S. Army TACOM Mobility Systems Laboratory Technical Report No. 11900 (LL-147) July 1974.
- 3) Karafiath, L.L. and Sobierajski, F.S., "Effect of Speed on Tire-Soil Interaction and Development of Towed Pneumatic Tire-Soil Model," U.S. Army TACOM Mobility Systems Laboratory Technical Report No. 11997 (LL-151) Oct 1974.
- 4) Nowatzki, E.A. and Karafiath, L.L., "The Effect of Cone Angle on Penetration Resistance," Grumman Research Department Memorandum RM-532J, Jan. 1972.

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account. The effect on various torque transfer mechanisms between the axles is also considered. The computer program for the vehicle performance model has been prepared as a subroutine with suitable arguments for use in the AMC Mobility Model. The vehicle performance model can be used with any soil, the strength of which can be characterized by its Coulomb strength parameters.

A braked tire-soil interaction model has also been developed for the estimation of the braking force that the vehicle can develop under various soil conditions.

A new method of analysis of the variation of cone penetration resistance with depth has been developed. In this method incremental penetration is analyzed by assuming that the stress state in the soil produced by the previous increment remains "locked in." Cone penetration resistance profiles can be converted to Coulomb strength parameters by this method using a trial and error procedure.

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